

GRADES 6-8

DISTRICT 742 COMMUNITY SCHOOLS

MATH PROGRAM GOALS

STANDARD ONE: NUMBER SENSE

Experience numbers in contextual situations and use them flexibly to solve problems, gradually moving to a more abstract understanding of number concepts, relationships and procedures.

GOAL 1: Middle/Grades 6-8:

Develop an understanding about numbers, their magnitude and their relationships.

- A. Develop benchmarks to comprehend the magnitude of large numbers.
- B. Develop benchmarks for common fractions.
- C. Develop an understanding of factors and multiples and of the relationships between them.
- D. Develop models for a variety of numbers such as prime and composite numbers, triangular and square numbers, square roots and powers/exponents.
- E. Name, estimate, compare and order fractions, decimals, percents, integers and rational numbers.
- F. Make comparisons among fractions, percents, ratios, rates and unit rates in appropriate situations.
- G. Develop and refine strategies for estimating quantities including fractional quantities and percentages.
- H. Estimate the reasonableness of probabilistic statements.
- I. Use rational numbers in a variety of ways: to label, measure, locate, compare and quantify.
- J. Use fractional models and symbolic representations of fractions to describe real-world situations.
- K. Solve problems using ideas about the multiplicative structure of numbers (primes, composites, factors, multiples and square numbers).
- L. Apply ratios, proportions and percents in a variety of situations.

M. Understand place value as used in a numeration system.

N. Create a real-world communication that demonstrates the ability to use a variety of numbers in context.

GOAL 2: Middle/Grades 6-8:

Develop an understanding of numeration systems and their properties.

A. Explore properties of the integer and rational number systems.

B. Explore number systems other than base 10, especially binary.

C. Explore the development of different number systems from historical and cultural perspectives.

D. Use scientific notation to represent very large and very small numbers.

E. Use fraction, decimal and percent forms to express rational numbers and move flexibly among these three forms of representation.

GOAL 3: Middle/Grades 6-8:

Translate among equivalent forms of numbers to facilitate problem solving.

Find equivalent fractions.

B. Identify location of fractions and decimals on a number line.

C. Represent and use fractions, decimals, percents, scientific notation, exponents and integers in mathematical situations.

D. Investigate the relationships among fractions, decimals and percents, and move flexibly among representations.

E. Use one- or two-dimensional graphs to depict numerical information and to interpret relationships (e.g. rates of change).

F. Use appropriate technologies to depict, to describe and to enhance understanding of numerical information.

GOAL 4: Middle/Grades 6-8:

Develop operation sense, and use it to estimate results of operations.

A. Use manipulatives and other models to explain the results of operations on fractions, decimals, percents and integers.

B. Understand and apply the concept of a fraction as an indicated division (e.g., $\frac{3}{5}$ could be a model for $3 \div 5$).

C. Understand p as a ratio of circumference to diameter in any given circle.

D. Know when an estimate is appropriate and when an exact answer is

appropriate.

E. Determine whether a situation requires an overestimate or an underestimate.

F. Estimate computations with fractions, decimals, percents and integers (e.g., taxes, tips, interests, discounts).

G. Communicate and defend reasonableness and appropriateness of estimates.

H. Estimate the impact of scale change on objects.

I. Develop a sense for the behavior of exponential operations.

J. Solve a variety of problems by selecting appropriate operations.

GOAL 5: Middle/Grades 6-8:

Understand the operations, and apply them to solve problems.

A. Select and use appropriate methods for computing from among mental math, paper and pencil, calculator and computer methods.

B. Select efficient representations for rational numbers in order to simplify and solve problems.

C. Understand and apply the commutative, distributive and associative properties.

D. Interpret and operate on expressions involving p .

E. Interpret order of operations to accurately perform operations using a variety of technological tools.

F. Communicate strategies, and justify thinking in solution processes.

G. Solve a variety of problems by generating and describing more than one method to solve problems.

H. Given problems with proposed solutions, analyze and justify operations and methods used.

STANDARD TWO: SHAPE, SPACE AND MEASUREMENT

Develop visualization and reasoning skills by investigating shape, space and measurement concepts and axiomatic systems, using appropriate tools and technologies.

GOAL 1: Middle/Grades 6-8:

Recognize, describe and model shapes and relationships, and represent and reason about them in increasingly abstract ways.

- A. Construct and model 2- and 3-dimensional figures from various points of view using appropriate tools and technologies (i.e. software like Geometric Pre-Supposer, Building Perspectives, Super Factory, etc.).
- B. Recognize critical attributes of shapes in the physical world (e.g., classify a square as a special type of rectangle).
- C. Describe, compare and contrast shapes and solids in mathematical language, using their essential properties.
- D. Develop and use definitions, and present informal arguments based on properties and relationships of shapes.
- E. Apply the concept of proportionality to solve problems involving congruence and similarity.

GOAL 2: Middle/Grades 6-8:

Use transformations and concepts of location, relative position and symmetry to visualize, represent and verify geometric relationships.

- A. Describe the location of objects by using Cartesian coordinates.
- B. Apply scale factor to transform geometric shapes or designs on a coordinate plane (e.g., use scale factors to enlarge a cartoon).
- C. Use drawing tools or geometric software (e.g., Kali, Tesselmania, Geometric Golfer, Kaleidotile) to create transformations, tilings and designs.
- D. Identify symmetries in 2- and 3-dimensional figures.
- E. Recognize and describe shape, size and position of 2- and 3-dimensional objects and their images under transformations.

GOAL 3: Middle/Grade 6-8:

Use measurement to make the connection between number and space in order to describe, analyze and represent real-world and abstract situations.

- A. Estimate measures with a specified degree of accuracy, and decide if an estimate or measurement is "close enough."
- B. Apply proportional reasoning (and a reference table when necessary) to convert units within a given measurement system.
- C. Apply proportional reasoning to solve problems involving indirect measurement.
- D. Apply scale factor to linear dimensions, and describe relationships to

resulting measures of perimeter, area and volume.

E. Measure length, mass, perimeter and area of quadrilaterals and circles; surface area and volume of solids; angles; determine type of measurement needed; select tools and units; measure to appropriate accuracy.

F. Describe how changes in the dimensions of figures affect perimeter, area and volume.

STANDARD THREE: DATA INVESTIGATION

Ask questions, and collect, organize, interpret and transform data related to those questions to communicate information and make predictions and decisions.

GOAL 1: Middle/Grades 6-8:

Formulate a question(s), determine necessary data and choose an appropriate method of data collection in order to make sense of a problem or situation.

A. Identify a problem of interest that moves beyond the individual or school to the larger community.

B. Work in small groups to clarify questions to focus the investigation and the data collection plan.

C. Refine language to clearly communicate intent of survey questions.

D. Critique questions generated by the class or posed in the media.

E. Design an investigation plan, including appropriate technology, to collect data.

F. Choose an appropriate sample.

G. Identify target audience, and create a data investigation plan to support an opinion or argument (e.g., "more people carpool to work").

GOAL 2: Middle/Grades 6-8:

Collect, organize and represent data in a variety of ways.

A. Access data from reference sources as basis for some investigations.

B. Explore various methods of displaying data, including cumulative frequency distributions, scatter plots, stem and leaf plots, line graphs, circle graphs and spreadsheets.

- C. Select an appropriate method to display data given the type of data collected and the kinds of comparisons to be made.
- D. Use technical tools (calculators and software) to construct graphical displays appropriate to the data.
- E. Determine and represent appropriate intervals on graphic displays.
- F. Explore sorting data in a database.
- G. Use telecommunications to obtain data from populations outside one's community.

GOAL 3: Middle/Grades 6-8:

Read, describe and interpret displays of data.

- A. Describe factual information shown on a graph.
- B. Calculate measures of center (mean, median, mode) from a display of numerical data.
- C. Determine and explain which measure of center best describes what is typical about the data.
- D. Explore measures of variability (range, quartiles, percents, percentiles, outliers), and discuss their effect on the shape of the data.
- E. Use scatterplots to explore the simple correlation between two sets of data.
- F. Use scatterplots and coordinate graphs of bivariate data to begin exploring the concepts of dependent and independent variables.
- G. Identify trends in data collected over time and differences across various populations.
- H. Analyze data by selecting and applying appropriate data measurement concepts.

GOAL 4: Middle/Grades 6-8:

Formulate, justify and communicate conclusions, arguments, predictions, decisions and further investigations based on data.

- A. Discuss which possible conclusions are supported by the data.
- B. Select the information, and determine an appropriate presentation method for a given purpose or a given audience.
- C. Discuss the impact the data might have on decision-making, depending on the slant of the interpretation or the population(s) surveyed.

- D. Discuss the degree of confidence students have in their results.
- E. Recognize the cyclical nature of investigations (i.e., results tend to lead to new questions).
- F. Use appropriate terminology to communicate an analysis of the data orally and in writing.
- G. Take appropriate action based upon the results of the data analysis.
- H. Explore possible sources of bias in data investigation plans.
- I. Summarize and critique data investigations done by others.

STANDARD FOUR: PROBABILITY, RANDOMNESS AND UNCERTAINTY

Apply concepts of probability, randomness and uncertainty to make critical judgements, predictions and decisions.

GOAL 1: Middle/Grade 6-8:

Develop an intuitive sense of probability, use probability as a measure of uncertainty, and develop and use the language of probability to communicate.

- A. Estimate the likelihood of future events on a scale of 0-1.
- B. Use rational numbers to quantify results of probability experiments or simulations.
- C. Understand the concept of a random event, and recognize that probability describes the long-run frequency of a random event, not what will happen in a single trial.
- D. Describe events, and give qualitative and quantitative descriptions of their likelihood.
- E. Explore the variables which affect the possible outcomes of an event, including exploration of intuitive approaches to dependence, independence and conditional probability.

GOAL 2: Middle/Grades 6-8:

Model situations involving uncertainty by designing and carrying out experiments or simulations to estimate probabilities, solve problems and make predictions.

- A. Discuss, devise and carry out a simulation of a situation using devices such as cards, dice and spinners; collect data and use computer simulation when appropriate to extend the number of trials.
- B. Explore strategies to determine all possible outcomes (e.g., lists, tree

diagrams, area models).

C. Make inferences and convincing arguments based on relative frequency, and predict results of further experiments.

D. Explore area models to estimate theoretical probabilities.

E. Make predictions based on theoretical probability.

GOAL 3: Middle/Grades 6-8:

Create and use different types of distributions as a basis for making inferences or predictions about outcomes or populations.

A. Examine and explain how the results from a simulation or experiment tend to better approximate the theoretical probability distribution as the number of trials increases.

B. Analyze a probability distribution to predict future outcomes.

C. Compare results from multiple simulations to form a sampling distribution.

D. Explore how the spread and shape of a sampling distribution changes as the sample size increases.

E. Organize raw data, and represent it in more than one way.

F. Analyze data by selecting and applying appropriate data-measurement concepts.

GOAL 4: Middle/Grade 6-8:

Explore current problems and events involving uncertainty, and recommend or justify a course of action, when appropriate.

A. Develop an appreciation for the pervasive use of probability in the real world.

B. Conduct a probability experiment to simulate a real-life issue involving uncertainty; use the results to recommend and communicate a course of action or make an informed decision.

C. Make critical judgments about claims made in probabilistic situations.

D. Begin to examine statements made about probabilistic situations such as weather forecasts, medical research, environmental risks.

E. Formulate a question, and design an appropriate data investigation.

STANDARD FIVE: PATTERNS, RELATIONSHIPS AND FUNCTIONS

Represent, compare and analyze mathematical patterns, relationships and functions to model and solve problems.

GOAL 1: Middle/Grades 6-8:

Recognize, describe and generalize patterns, and build mathematical models to make predictions.

- A. Represent mathematical relationships with tables, graphs in the coordinate plane and verbal or symbolic rules.
- B. Communicate patterns verbally and in writing.
- C. Analyze rational number patterns.
- D. Use a variety of materials to explore patterns to generalize perimeter, area, surface area and volume formulas.
- E. Apply patterns to explore number theory (e.g., figurate numbers, Fibonacci sequence, Pascal's Triangle).
- F. Given a sequence, generate a rule; given a rule, generate a sequence.
- G. Recognize the use of patterns in other disciplines and in a variety of cultures.
- H. Use appropriate technology to informally explore patterns of growth and be able to generalize these observations into some frequently occurring types of familiar functions.
- I. Use patterns to solve problems.

GOAL 2: Middle/Grades 6-8:

Analyze the interaction between quantities and/or variables to model patterns of change.

- A. Explain how a change in one quantity results in a change in another.
- B. Differentiate between and explain in an informal way types of changes in patterns (e.g., linear vs. non-linear, direct vs. indirect, continuous vs. non-continuous).
- C. Use one or more variables to represent patterns of change.
- D. Represent variability or change in a real-world situation using ordered pairs, tables, graphs and equations.
- E. Predict and describe mathematical relationships in real life "messy" data (e.g., measuring shoe size to height).
- F. Given a graph, tell a story; given a story, sketch a graph.

G. Explore the concept of rate of change in a linear function.

H. Use technological applications (including spreadsheets and graphing utilities) to examine cause and effect relations in functions.

GOAL 3: Middle/Grades 6-8:

Use algebraic concepts and processes to represent and solve problems that involve variable quantities.

A. Represent algebraic concepts with geometric models (e.g., algebra tiles) and physical models (e.g., balance scale, tables and graphs).

B. Interpret, write and evaluate algebraic expressions connected to real-life situations or story problems.

C. Express patterns in more general or abstract terms; formalize a rule that describes a pattern of change.

D. Use concrete, informal and formal methods to solve linear equations and inequalities; explain solution(s) in the context of the problem.

E. Develop concepts, vocabulary and symbols for variable, expression, equation, equality, inequality, replacement set and ordered pairs.

F. Use a variety of appropriate representations to describe relationships between/among quantities in problem situations.

STANDARD SIX: DISCRETE MATHEMATICS

Use discrete mathematics to model and understand concepts and relationships and to solve problems.

GOAL 1: Middle/Grades 6-8:

Investigate and apply systematic counting techniques, set relationships and principles of logic to represent, analyze and solve problems.

A. Use manipulatives, diagrams or systematic lists to develop counting strategies and apply them in appropriate situations.

B. Interpret set relationships and notation from real-world contexts to represent algebraic concepts.

GOAL 2: Middle/Grades 6-8:

Use charts, vertex-edge graphs and matrices to model and solve problems.

A. Extend tree diagrams to represent factors of a number and outcomes in a probability experiment.

B. Use a systematic approach to identify, represent and record all possible paths or outcomes in an experiment.

C. Design vertex-edge graphs to represent relationships between or among objects.

D. Investigate some classical graph theory problems (for example: highway inspector problem, garbage collection problem, traveling salesperson problem).

GOAL 3: Middle/Grades 6-8:

Explore, develop and analyze algorithmic thinking to accomplish a task or solve a problem.

A. Use manipulatives, drawings or descriptions to demonstrate algorithmic thinking.

B. Explain the output of an algorithm described by a flow chart.

C. Create, communicate and defend a strategy for winning a game.

D. Create and communicate algorithms to solve problems.

GOAL 4: Middle/Grades 6-8:

Analyze, extend and model iterative and recursive patterns.

A. Explore patterns in more complex geometric and numerical problems (e.g., Fibonacci sequence, Pascal's triangle).

B. Examine patterns in an iteration to predict successive terms and long-range trends.

C. Employ recursion in counting contexts (e.g., moves in Tower of Hanoi puzzle).